**SETON HALL UNIVERSITY**

**TWENTIETH ANNUAL**

**JOSEPH W. ANDRUSHKIW**

**MATHEMATICS COMPETITION**

1. Betty weighs twice as much as her sister Emily and 10 pounds more than her cousin Kate. The sum of the weights of Betty, Emily and Kate is 210 pounds. Find Kate's weight. (Answer: 78)

2. The measures of the angles of a (convex) pentagon are in the ratio 3:4:4:4:5. Find the degree measure of the largest angle in the pentagon. (Answer: 135)

3. Dave has \$16.00 in stamps. He has only 20¢-stamps, 37¢-stamps and 60¢-stamps. He has at least six 20¢-stamps , at least ten 37¢-stamps, and at least eight 60¢-stamps. What is the smallest number of stamps he can have? (Answer: 39)

4. Find the positive integer base *B* such that $123_4 + 135_6 + 147_8 + 159_{10} = 183_B$. (Answer: 15)

5. Find the units digit of the number $1^{101} + 2^{102} + 3^{103} + 4^{104} + 5^{105} + 6^{106} + 7^{107} + 8^{108} + 9^{109}$ (when expressed as an integer in decimal form). (Answer: 7)

6. The integers $N_k$ are defined by $N_k = 1 + 4 + 7 + \cdots + (3k+1)$, for $k$ = 1, 2, 3, … . (Therefore $N_1 = 1+4, N_2 = 1+4+7, N_3 = 1+4+7+10,$ and so forth.) If $N_{m+4} - N_m = 274$ (where $m$ is a positive integer), find $N_{m+8} - N_m$. (Answer: 596)

7. Sixty young adults were questioned regarding their participation in the sports of football (F), basketball (B), volley ball (V), and track (T). It was found that two had participated in all four sports; 6 in F, B, and V; 5 in F, B, and T; 5 in F, V, and T; 6 in B, V, and T; 11 in F and B; 14 in F and V; 12 in F and T; 15 in B and V; 11 in B and T; 11 in V and T; 28 in F; 25 in B; 29 in V; 23 in T. Find the number that participated in none of the four sports. (Answer: 9)

8. A 5-digit number is to be formed using any of the digits 1, 2, 3, 4, or 5 any number of times. (Repetition of digits is allowed). The tens' and hundreds' digits are to be both even or both odd, and the thousands' digit is to be greater than the units' digit. How many such 5-digit numbers can be formed? (Answer: 650)

9. Let region $R$ consist of points (x, y) on a coordinate plane such that $x$ and $y$ are both integers, $y^2 - x^2 \geq 15$ and $y^2 + 2x^2 \leq 36$. Find the probability that if a point (x, y) in region $R$ is chosen at random, its coordinates satisfy the condition $4x^2 > 9$. (Answer: 2/9)

10. If $v$ is the complex number $-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i$ and $w$ is the complex number $\frac{1}{2} - \frac{\sqrt{3}}{2} i$, find $v^{64} + w^{84}$.
(Answer:2)

11. In a circle with center O, radii OC and OD form an angle of $t$ radians ($t$ real, $t < \pi$). The area of the segment of the circle enclosed by chord CD and (smaller) arc CD is equal to the area of triangle COD. If $\sin(t) = k$, where $k$ is a positive real number , find $k$ in terms of $t$. (Answer: t/2)

12. Let $x$ and $y$ be real numbers for which $0 < x < y$. Find the value of $y$ for which
$\log_9\left(x^3 + y^3\right) - \log_9\left(x^4 + y^4\right) + \log_9\left(x^4 y^3 + y^7\right) - \log_9\left(x^2 + 2xy + y^2\right) +$
$\log_9\left(x + y\right) - \log_9\left(x^2 - xy + y^2\right) + 3/2 = 0.$ (Answer: 1/3)

13. Points A = (-20, 20) and B = (23, -12) lie on a coordinate plane. Find all positive real numbers $t$, given that the point C = ($t$, 0) lies on this coordinate plane and is 4 times as far from A as from B.
(Answer: 28 and 356/15)

14. Al can do a certain piece of work (alone) in 10 days. Bob can do the same piece of work (alone) in $N$ days (where $N$ is a positive real number). Al works alone until the work is 1/4 finished; then Al and Bob work together for 4 days to complete the work. Find the total number of days required to complete a piece of work of this type, if Bob works alone until the work is 1/3 finished, then Al and Bob work together to complete the work. (Answer: 464/63)

15. Ron will take a trip by train and make connections at stations $s_1, s_2, s_3,$ and $s_4$. Each time he makes a good connection at $s_1$ or $s_2$ or $s_3$, the probability of making a good connection at the next station is ¾ (thus the probability of not making a good connection is ¼); each time he makes a bad connection at $s_1$ or $s_2$ or $s_3$ the probability of making a bad connection at the next station is $3/5$ (thus the probability of not making a bad connection is $2/5$ ). Assume that the probability of a good connection at $s_1$ is $9/10$ (and of a bad connection is $1/10$). Find the probability that Ron had exactly two bad connections at $s_1, s_2, s_3,$ and $s_4$. Give the answer in reduced rational form. (Answer: 829/4000)

16. Quadrilateral ABCD of perimeter 684 inches is inscribed in a circle; the lengths of sides AB, BC, CD and DA form an arithmetic progression with $\overline{AB} < \overline{BC} < \overline{CD} < \overline{DA}$. If the secant of angle BAD has value

89, find the length of the largest side of quadrilateral ABCD. (Answer: 198)