# SETON HALL UNIVERSITY

# TWENTYFIFTH ANNUAL

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# MATHEMATICS COMPETITION

1. The sum of the squares of four consecutive positive integers is 1374, Find the smallest of the four consecutive integers. **ANS: 17**

2. Find the smallest real number, $x$, for which $\frac{2x^2+3x}{3x^2-10x+8} \le 0$. **ANS: -3/2**

3. Let L($N$) denote the sum of the smallest and the largest prime factors of the integer $N$, $N > 2$. For example L(750) = 5 (since $750 = 2 \cdot 3 \cdot 5^3$ and 2 + 5 = 7) and L(81) = 6 (since $81 = 3^4$ and 3 is both the smallest and largest prime factor of 81 so that L(81) = 3 + 3 = 6). Find the smallest four-digit positive integer $N$ for which L($N$) = 4. **ANS: 1024**

4 A committee composed of either 4 or 5 members is to be formed; members from 5 in group $A$, 3 in group $B$ or 4 in group $C$. At least one from group $B$ must be chosen and at most 3 from any of the three groups may be chosen. How many such committees can be formed? **ANS: 1017**

5. The horizontal line with equation $y$ = -14 is tangent to the graphs (on a coordinate plane) of both $y = 5\cos(2x) - 3\sin(x) + 3c$ and $y = 4\csc(x-\pi) + 5c$; where $c$ is an integer. Find $c$. **ANS: -2**

6. Express the number $N = .472397239... = .4\overline{7239}$ in rational form (i.e. in the form $\frac{n}{m}$ where $n$ and $m$ are positive integers), reduced to lowest terms. **ANS: 3149/6666**

7. Four integers are randomly chosen from the set $S$ = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} without replacement. Find the probability that the four integers chosen (when arranged in order of size) form an arithmetic progression. **ANS: 2/35**

8. Consider equations I: $x^2 + bx + c = 0$ and II: $x^2 + dx + f = 0$, where $b$, $c$, $d$, and $f$ are nonzero real numbers. The discriminant of equation I is 5. The roots of equation I are triple the roots of equation II. Find the discriminant of equation II. **ANS: 5/9**

9. Find all ordered triples ($x$, $y$, $z$) of real numbers which simultaneously satisfy the equations:

$$\begin{aligned} x^2 + y^2 - z^2 &= 1 \\ x + y - 5z &= 2 \\ 2x - y + 2z &= 1 \end{aligned}$$

**ANS: (1/2,-1,-1/2), (7/8,1/2,-1/8)**

10. Right triangle $QP_0P_4$ has base $QP_0$ and right angle $QP_0P_4$. Side $P_0P_4$ is divided into four segments by points $P_1, P_2, P_3$ with $P_1$ between $P_0$ and $P_2$, $P_2$ between $P_1$ and $P_3$, $P_3$ between $P_2$ and $P_4$. $\overline{QP_0}$ is 240 feet long, $\overline{P_0P_4}$ is 180 feet long. Denote the area of triangle $QP_0P_1$ by $A_1$, of triangle $QP_1P_2$ by $A_2$, of triangle $QP_2P_3$ by $A_3$ and of triangle $QP_3P_4$ by $A_4$. If $A_4$ exceeds $A_2$ by 7680 ft$^2$, $A_3$ exceeds $A_2$ by 1680 ft$^2$, and $A_1$ exceeds $A_2$ by 4560 ft$^2$, find the area and perimeter of triangle $QP_2P_3$. **ANS: Per 540 ft, Area 3600 ft$^2$**

11. Find all positive real numbers $x$ which satisfy the equation $64(\log_{16} x)^4 + 136(\log_{16} x)^3 + 86(\log_{16} x)^2 + 11(\log_{16} x) - 3 = 0$. **ANS: 1/16, 1/4, 1/8, $\sqrt{2}$**

12. Consider the complex numbers 1 + $i$ and $\frac{-\sqrt{3}}{2} - \frac{1}{2}i$, where $i^2 = -1$. Find the largest positive 2-digit integer $N$ for which both $(1+i)^N$ and $\left(\frac{-\sqrt{3}}{2} - \frac{1}{2}i\right)^N$ are negative integers. **ANS: 84**

13. The ellipse $16x^2 + 25y^2 = 400$ and the parabola $y^2 = 12x$ lie on a coordinate plane and intersect in two points, $A$ and $B$. Find an equation of the circle with center at the origin which passes through points $A$ and $B$. **ANS:** $x^2 + y^2 = 265/16$

14. A train goes from $A$ to $B$ to $C$ to $D$, a distance of 319 miles. The distance from $A$ to $B$ is 33 miles more than the distance from $B$ to $C$ and the distance from $B$ to $C$ is 11 miles more than the distance from $C$ to $D$. On "Slow Day", the train travelled at rate $r_1$ from $A$ to $B$, than at a rate ($r_2$) half of $r_1$ from $B$ to $C$, and at a rate ($r_3$) two-thirds of $r_1$ from $C$ to $D$. The usual rate for the entire trip is $r_1$ . It took 13/6 hours longer on "Slow Day" from $A$ to $D$ then it usually takes. Find the rate $r_1$ . **ANS 66 mph**

15. Let $P_1 = x+1$, $P_2 = (x+1)(x^2-1)$ and $P_3 = (x+1)(x^2-1)(x^3+1)$, where $x$ is a real number and not an integer. Find a rational number $x$ which is a solution of the equation $\frac{x}{P_1} - \frac{P_1(x-1)}{P_2} - \frac{P_2(x^3-1)}{P_3} = \frac{-100}{243P_3}$.

ANS: 2/3

16. Triangle $ABC$ has base $BC$ with points $D$, $E$ and $F$ on $BC$, $D$ between $B$ and $E$, $E$ between $D$ and $F$, and $F$ between E *and C*. Line segment $AD$ is perpendicular to side $BC$ at $D$, the length of side $AB$ is 2 feet, and the degree measure of each of the angles $BAD$, $DAE$, $EAF$, $FAC$ is 15°. By how many inches does the perimeter of triangle $AFC$ exceed the perimeter of triangle $AEF$? (Give the answer in exact form.)

**ANS:** $\left(\sqrt{2+\sqrt{3}}\right)\left(\sqrt{2} + 1 - \frac{2\sqrt{3}}{3}\right) + \sqrt{2-\sqrt{3}} - 2$